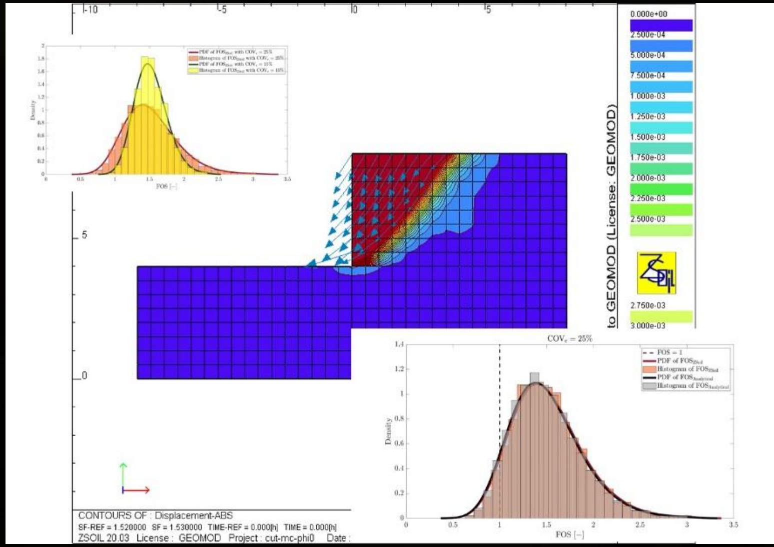
POLISH ACADEMY OF SCIENCES
GEOTECHNICAL DIVISION

Probabilistic approaches applied to geotechnical finite element analyses

Prof. Dr Stéphane Commend, iTec HEIA-FR HES//SO + GeoMod SA

April 8th, 2021



Outline

- Motivation: analytical and FE analysis of the stability of a vertical cut
- Uncertainty propagation, sensitivity and reliability analyses
- Example 1: 2D analysis of an excavation in urban environment
- Example 2: 3D settlement analysis of a concrete foundation
- Example 3: inverse analysis applied to a tunnel excavation in Paris
- Conclusion and perspectives

Motivation: analytical and FE analysis of the stability of a vertical cut

- Uncertainty is common in geotechnical engineering: the value of a given soil parameter (cohesion, friction angle, elastic modulus) is usually not known exactly, and it varies in space
- The common use is to test a small amount of samples at some locations, take the mean value of the set, carry a deterministic analysis and use (partial or global) "safety" factors in order to stay on the safe side. However, it doesn't give much insight into what the actual risk can be
- In reality uncertainty is present everywhere, in the assumptions made, in the timing of construction, in the geometry, in the material parameters, in the loads, and in the FE approximation

Motivation: analytical and FE analysis of the stability of a vertical cut

- Assume we analyze the stability of a H = 4 m high vertical cut in a purely cohesive medium
- Assume deterministic material parameters $\gamma = 20$ kN/m^3 and c = 30 kPa, constant over the domain
- We get SF = h_c/H equal to 1.50, approximately

[Broere, 2002]

2 STABILITY OF A VERTICAL CUT

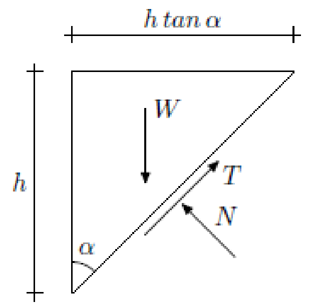


Figure 1. Definition of forces on a triangular failure body

for a purely cohesive material with cohesion c and from the last two equations it can be easily derived that

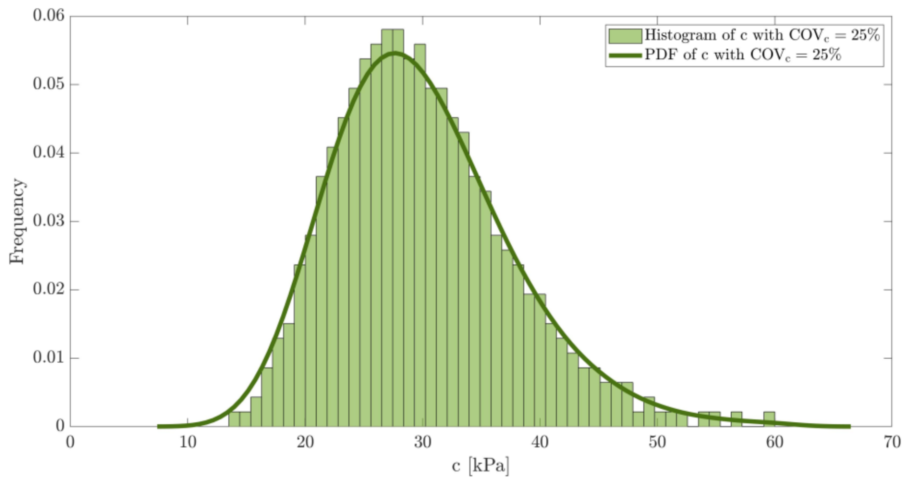
$$h = \frac{4c}{\gamma}\frac{1}{\sin 2\alpha} \quad (4)$$

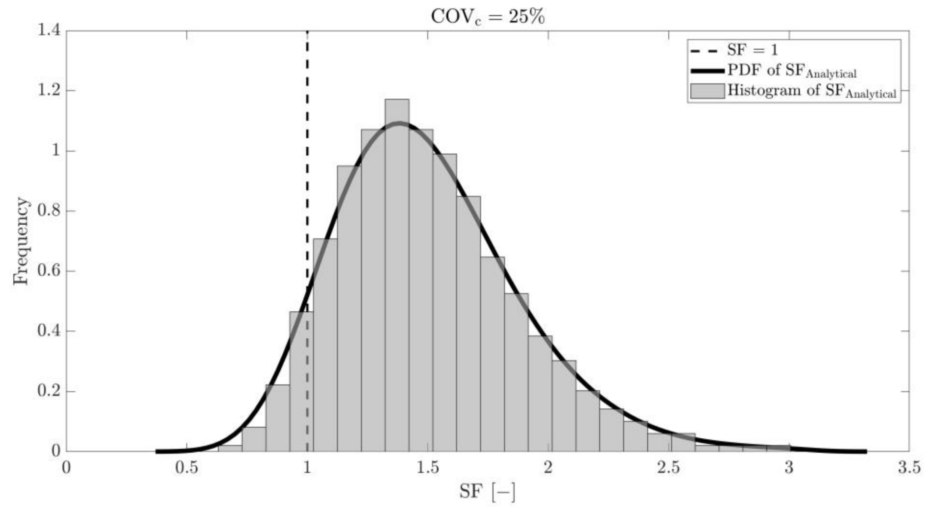
for a given slope α. Failure will occur for that value of α for which h is minimal, or $\alpha = 45°$. The critical height h_c is then found as

$$h_c = \frac{4c}{\gamma}. \quad (5)$$

Motivation: analytical and FE analysis of the stability of a vertical cut

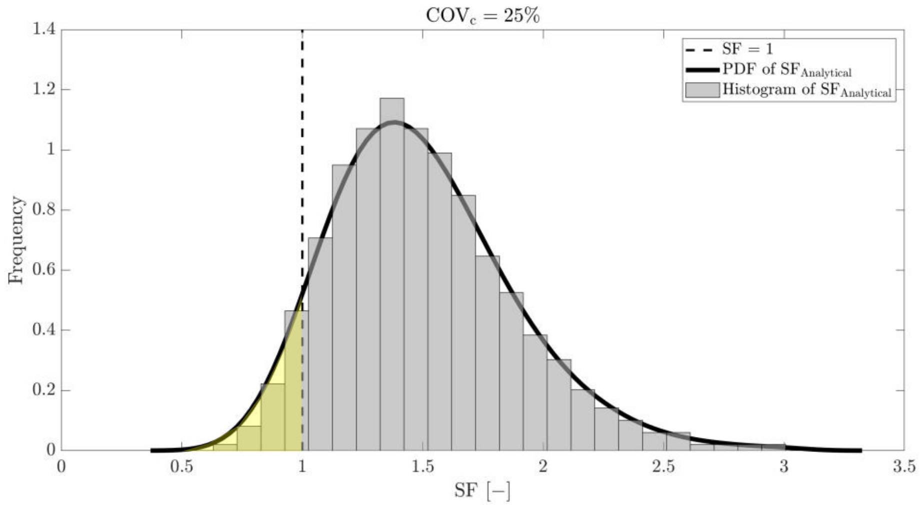
- Assume next that there is some uncertainty associated with c which varies around its mean value, with a coefficient of variation COV = 25%
- If we repeat the safety analysis 500 times with c varying randomly according to some credible statistical distribution (here lognormal), we will get a non negative distribution of SF values, with mean μ close to 1.50





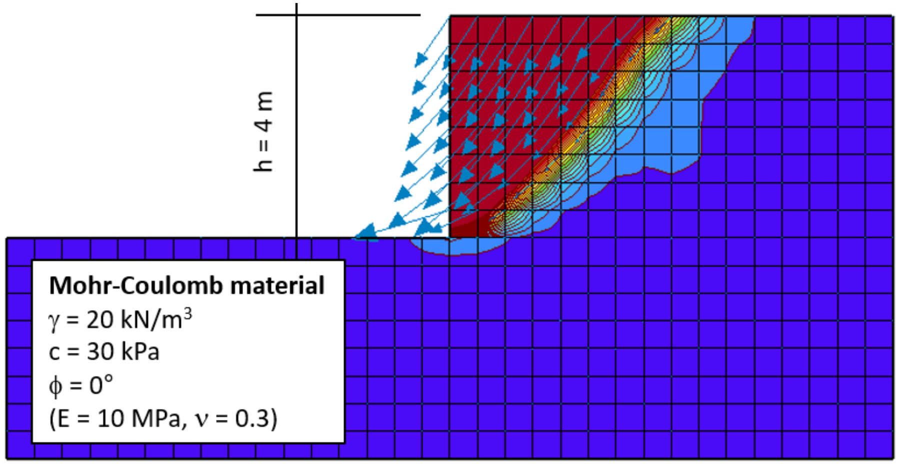
Motivation: analytical and FE analysis of the stability of a vertical cut

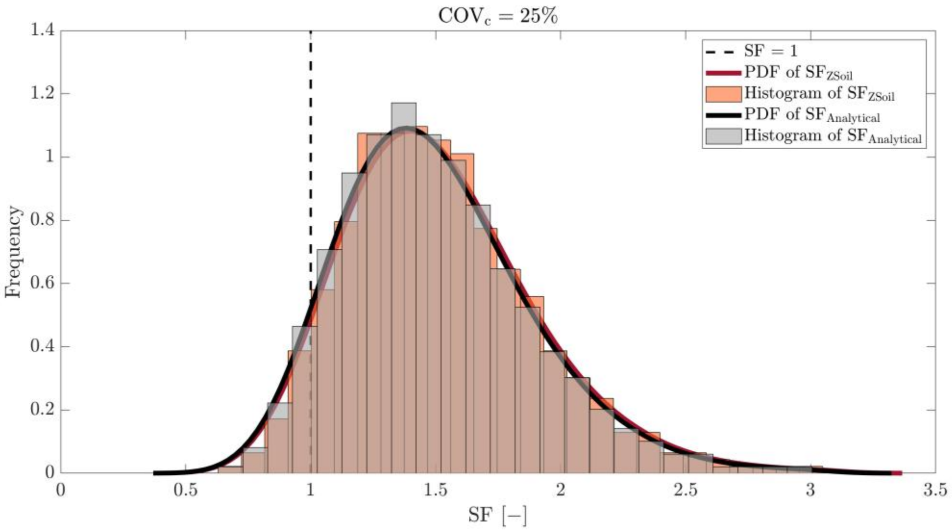
- The standard deviation in the output distribution of SF is a result of the uncertainty in the input value c. It is zero in the deterministic case and increases with uncertainty
- The probability of failure Pf that SF is smaller than 1.0 can then be computed by numerical integration of the PDF (probability distribution function) of SF between 0 and 1, as illustrated
- Also, a Monte-Carlo type procedure leads to Pf = #(SF<1) / #total



Motivation: analytical and FE analysis of the stability of a vertical cut

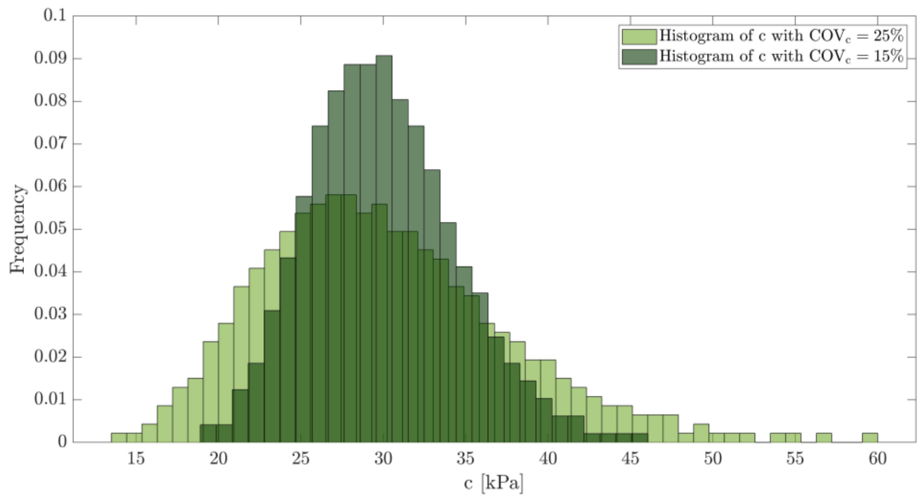
- The same example can be treated with a numerical approach, using the ZSOIL finite element software, with the same Monte-Carlo approach on 500 samples
- Note the good agreement between analytical and numerical PDF
- As a result, Pf(numerical) = 5.6%, close to Pf(analytical) = 6.2%

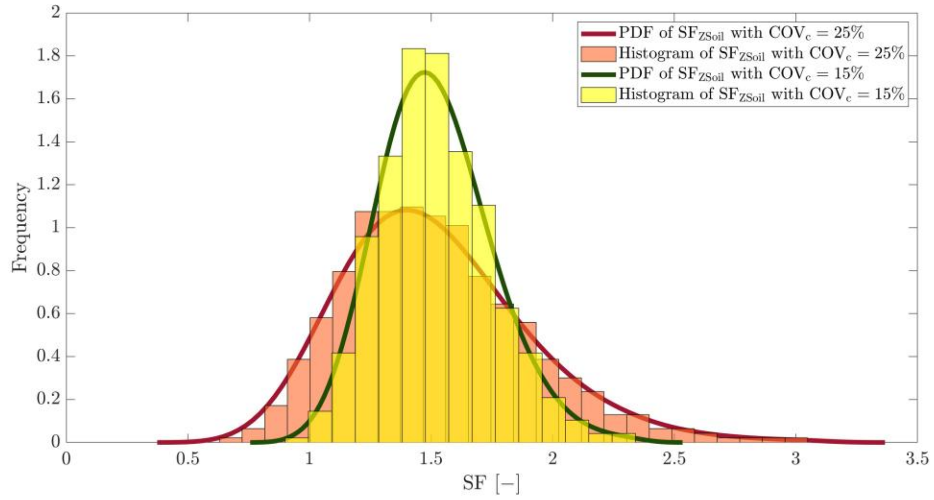




Motivation: analytical and FE analysis of the stability of a vertical cut

- If we vary the standard deviation value of c (say COV = 15% instead of 25%), the resulting SF distribution will vary
- As a consequence, Pf will also vary, even though the mean value for SF will remain close to SF = 1.50



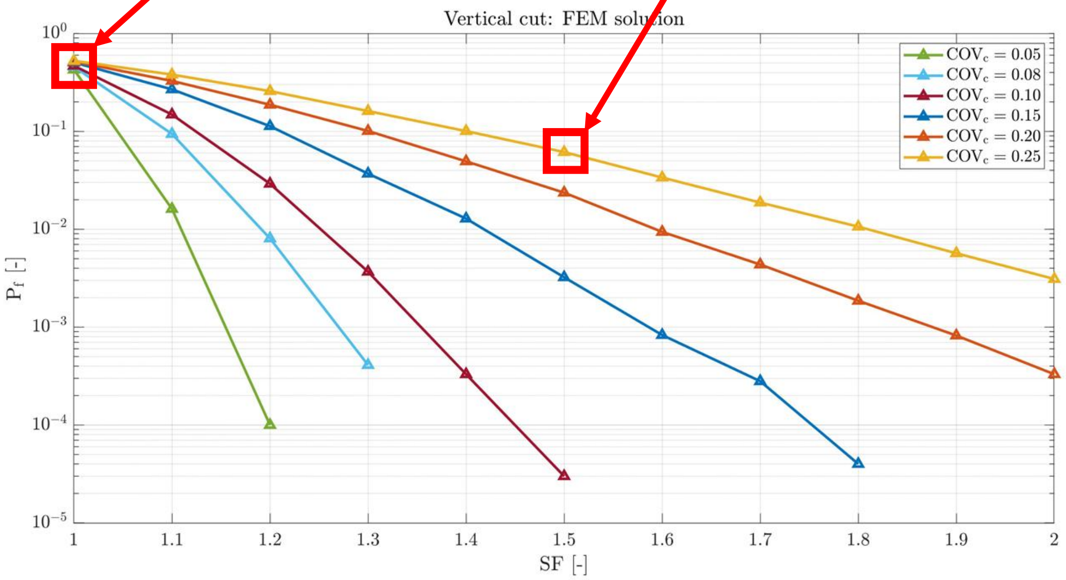


Motivation: analytical and FE analysis of the stability of a vertical cut

- If we vary next both the mean and the COV of c, both the mean and the COV of SF will vary, and as a consequence, Pf will vary as a function of SF
- It illustrates the fact that to achieve simultaneously acceptable values for SF, say SF > 1.50, and Pf, say Pf < 1e-3, uncertainty must be limited
- This may lead to additional testing e.g. or to a feedback procedure during construction
- Both SF and Pf are therefore of interest to civil engineers

when SF tends to 1, P_f converges towards a constant equal to the integral of the PDF between 0 and the peak

For μ(c) = 30 kPa with COV = 25%, SF = 1.50 and Pf = 0.06



Uncertainty propagation, sensitivity and reliability analyses

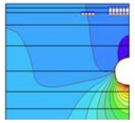
Deterministic analysis (or semi-probabilistic)

Input variables (*)

$\mathbf{X} = X_1, X_2, \ldots, X_n$

→

Model
- equation, MEF, etc.



→

Results (+)

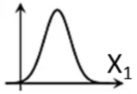
$\mathbf{Y} = Y_1, Y_2, \ldots, Y_n = f(\mathbf{X})$

(*) for example: cautious values for cohesion, friction angles, moduli of elasticity, groundwater table, loads, unloading rate etc.
(+) for example : a safety factor, the displacement of a given point, the bending moment in a retaining wall etc.

Probabilistic analysis

Quantifications of sources of uncertainty

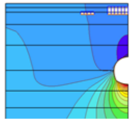
Probabilistic input variables (°)



→

Model definition

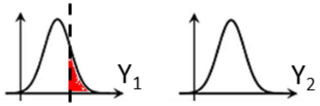
Model
- equation, FEM, etc.



→

Uncertainty propagation

Results: $\mathbf{Y} = f(\mathbf{X})$
1. Mean and standard deviation $\mathbf{Y}$ (μ, σ)
2. Probability density



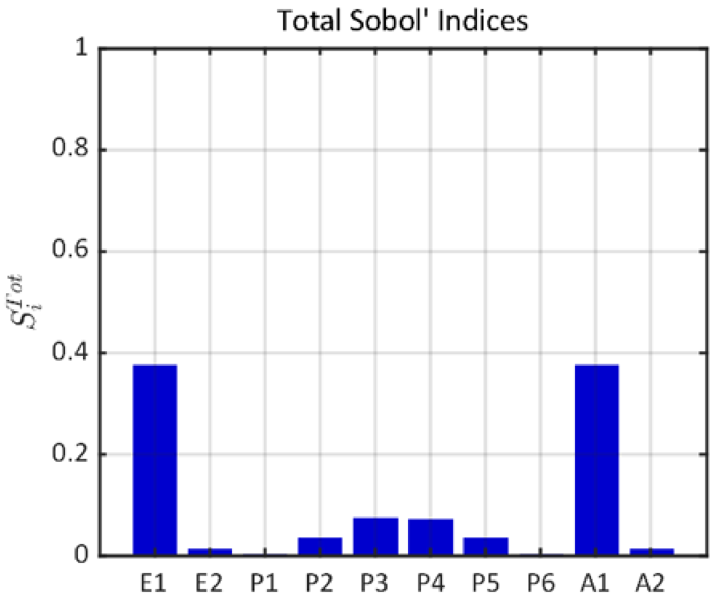
3. Probability of failure **Pf** or reliability index β

Sensitivity analysis, inverse analysis (Bayesian)

(°) represented by probability distributions (normal, lognormal, Gumbel, etc.) with mean μ and standard deviation σ

Uncertainty propagation, sensitivity and reliability analyses

- Sensitivity analysis aims at describing how the variability of the output is affected by the variability of each input variable
- Various methods, among them Sobol' indices
- Useful to help the engineer focus on the most impactful parameters

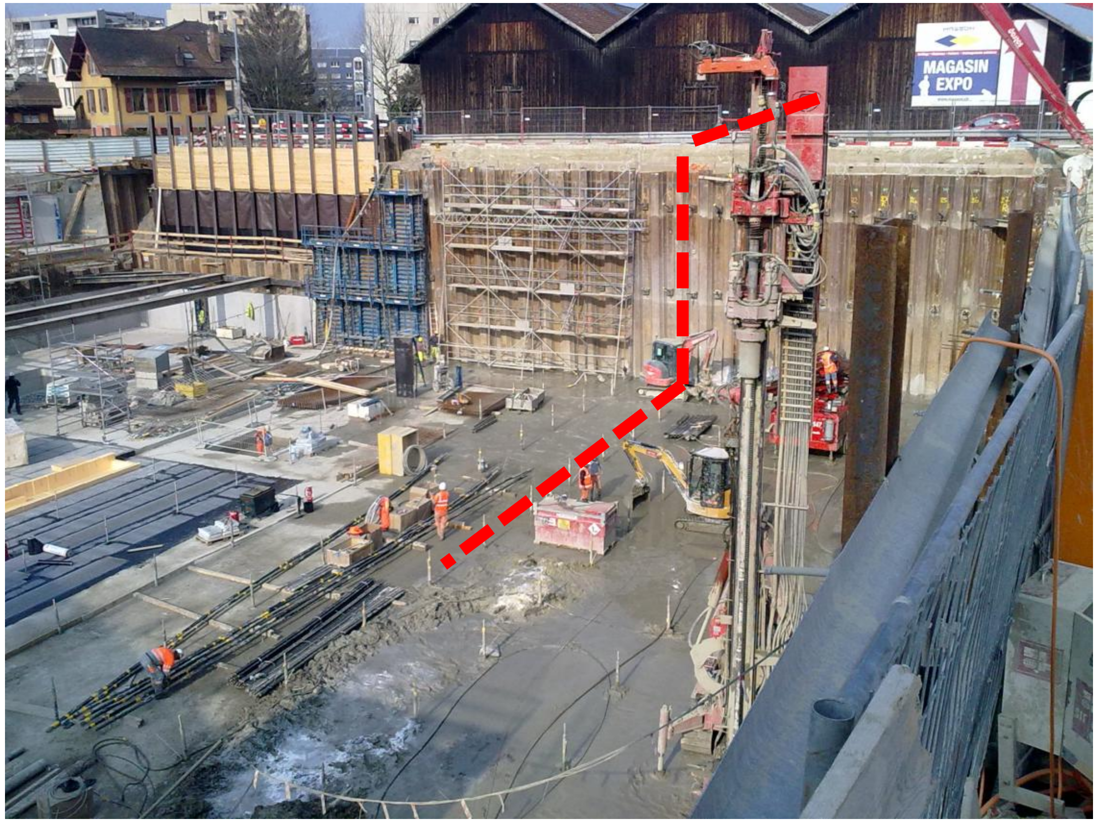


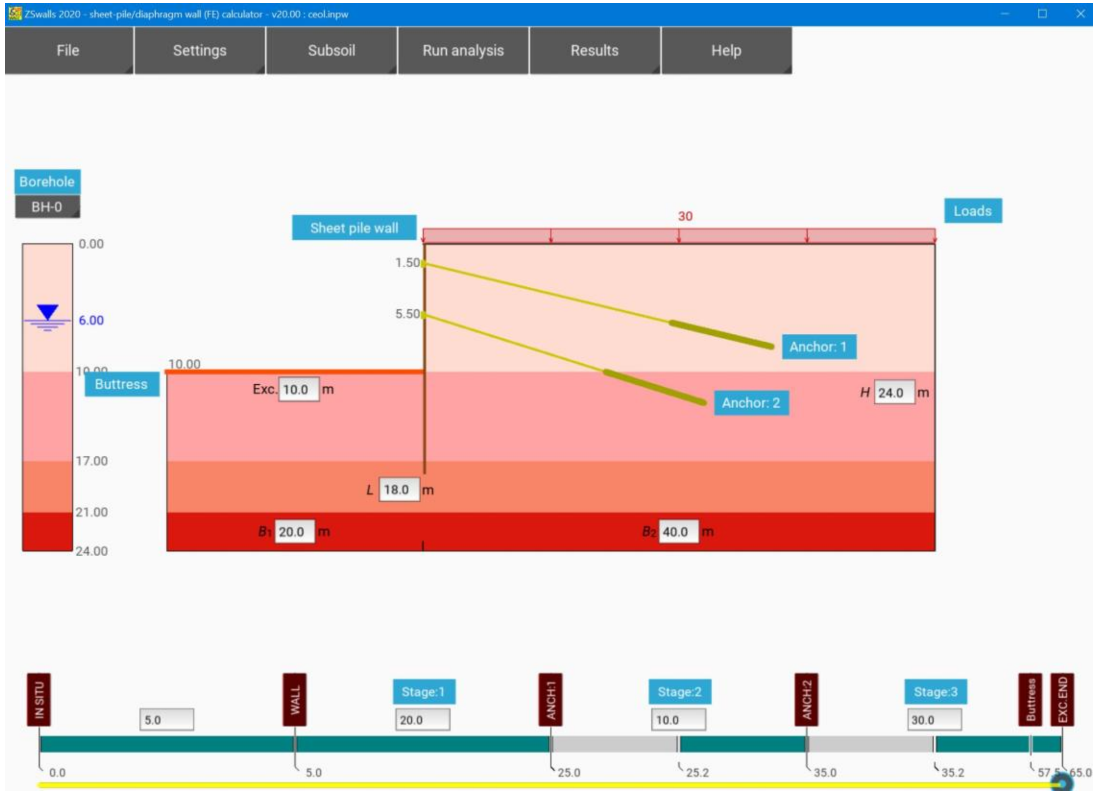
[www.uqlab.com]

Uncertainty propagation, sensitivity and reliability analyses

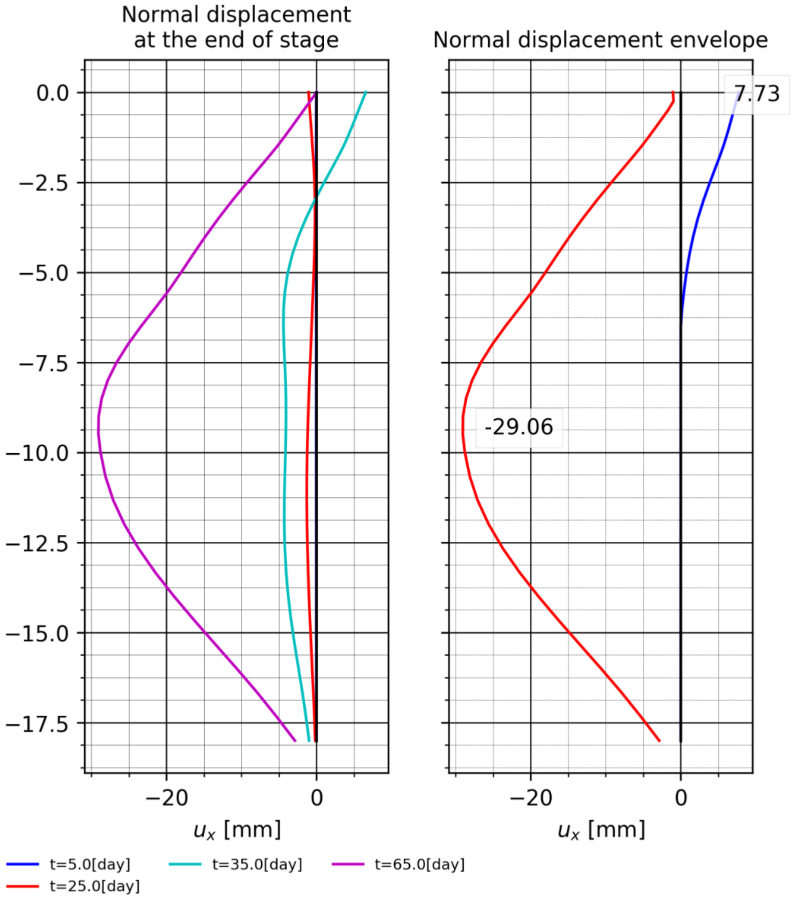
- Reliability analysis aims at finding the probability of failure (Pf) of the model given a failure function g(X)
- Examples:
  - g(X) = SF – 1.0 < 0
  - g(X) = threshold – settlement < 0
- Various algorithm exist to compute Pf (MCS, FORM, SORM, …)
- Can lead to large calculation time depending on the method chosen, especially for numerical modelling
- So… need for meta-models!
  - Adaptive Kriging-Monte Carlo (AK-MCS) is the most efficient if we're interested in Pf only (and not the PDF)
  - The polynomial chaos expansion (PCE) method is also of big interest

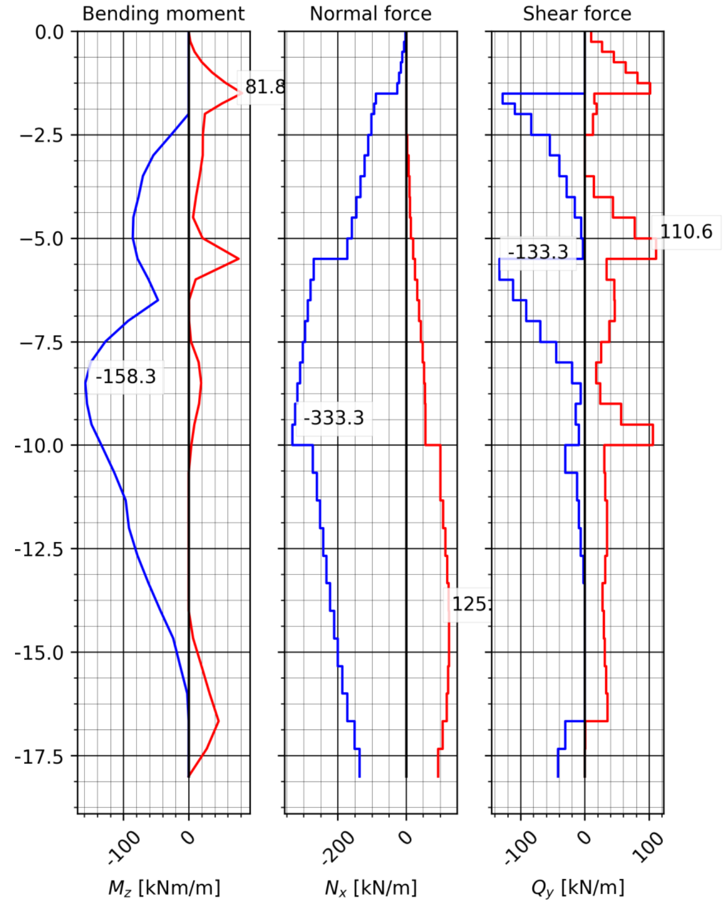
Example 1: 2D analysis of an excavation in urban environment





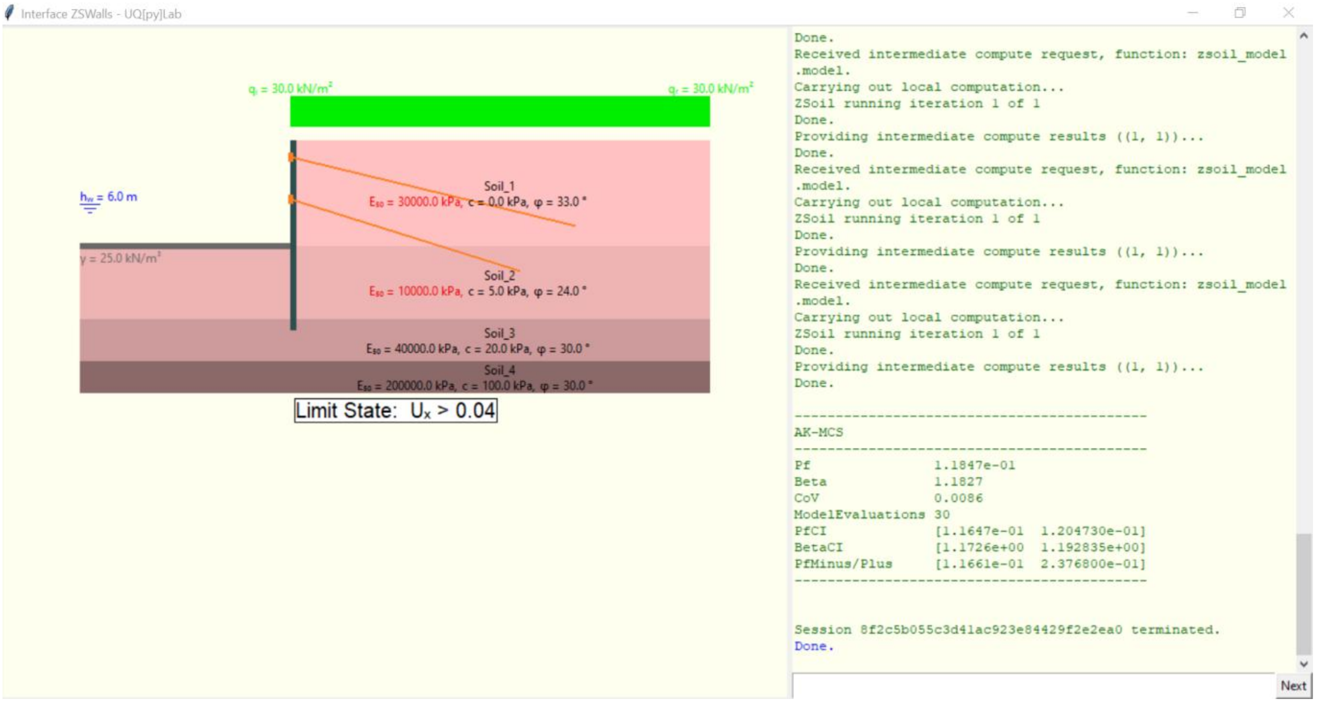
Example 1: 2D analysis of an excavation in urban environment





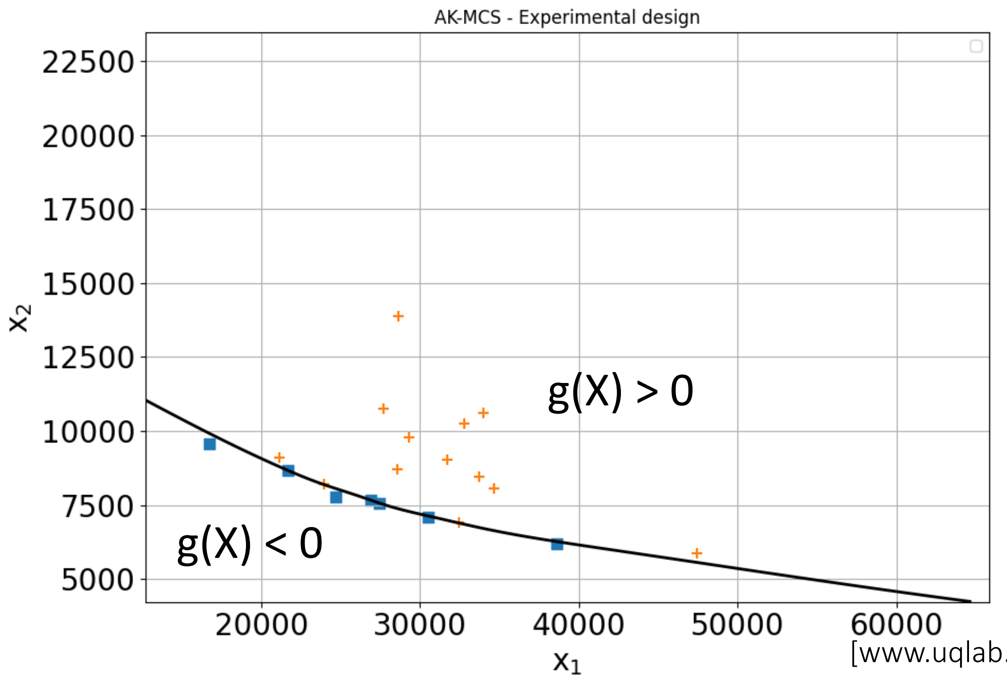
Example 1: 2D analysis of an excavation in urban environment

- Interface ZSWalls-UQLab
- E1 Young's modulus of the first soil layer: type = Lognormal, mean = 30 MPa COV = 20%
- E2 Young's modulus of the 2nd soil layer: type = Lognormal, mean = 10 MPa COV = 20%
- Limit state: ux > 40 mm
- Illustrate AK-MCS vs. PCE metamodels



Example 1: 2D analysis of an excavation in urban environment

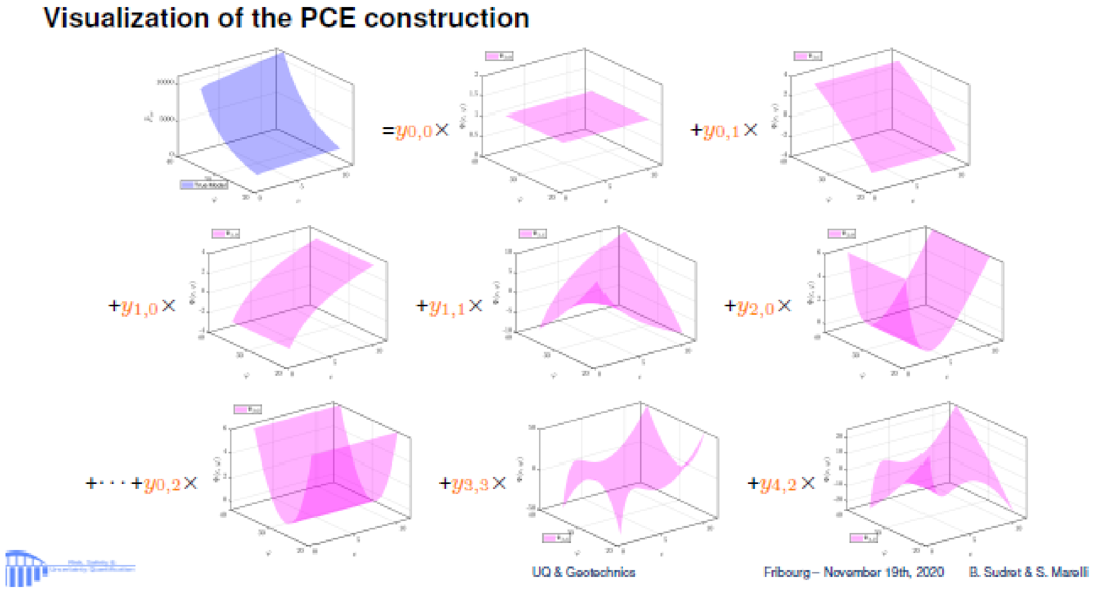
- AK-MCS method combines a Monte Carlo simulation with adaptive built Kriging meta-models to search points that lie close to g(X) = 0, reducing the total computational cost
- Here: 10 + 10 = 20 runs yields Pf(g(X) < 0) = 0.12



[www.uqlab.com]

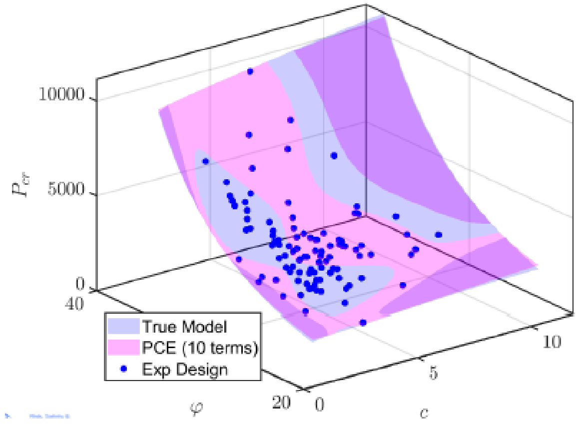
Example 1: 2D analysis of an excavation in urban environment

- PCE method consists in running a reasonable number of FEM simulations (between 10 and 100) and then, based on these simulations results, create a polynomial series which approximates the FEM
- Once this meta-model is built, it is easy to run a large number of calculations in a minimal time and thus the use of the MCS method is again possible within a reasonable calculation time



$$Y^{\text{PCE}} = \sum_{\alpha \in \mathcal{A}} y_{\alpha}\, \Psi_{\alpha}(X)$$

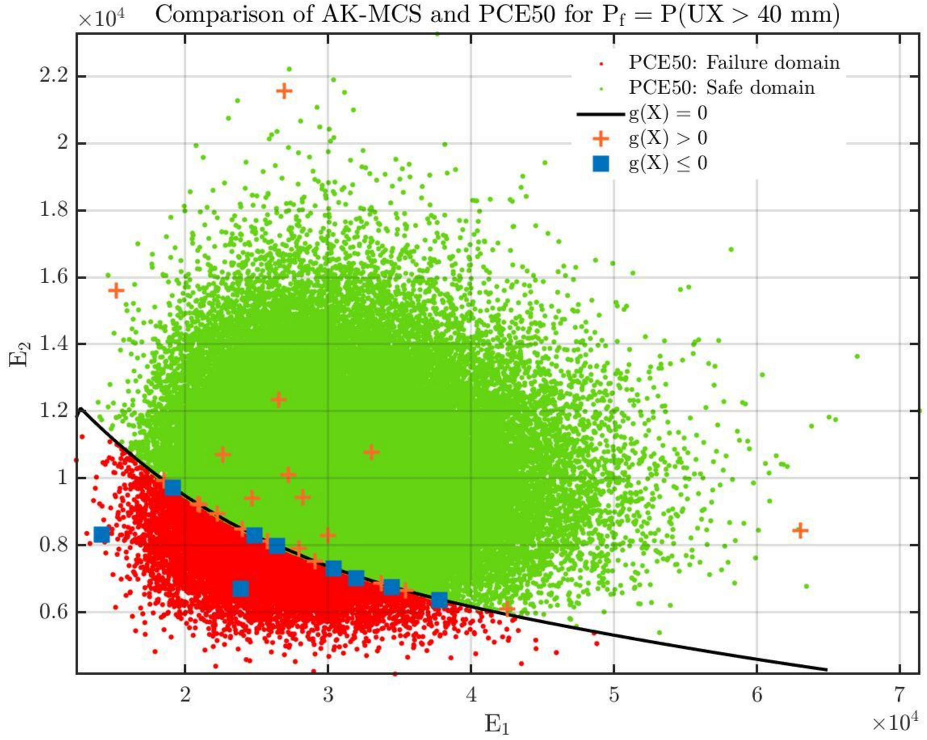
[www.uqlab.com]



Example 1: 2D analysis of an excavation in urban environment

P_f comparison for g(X) = 40 mm – UX.

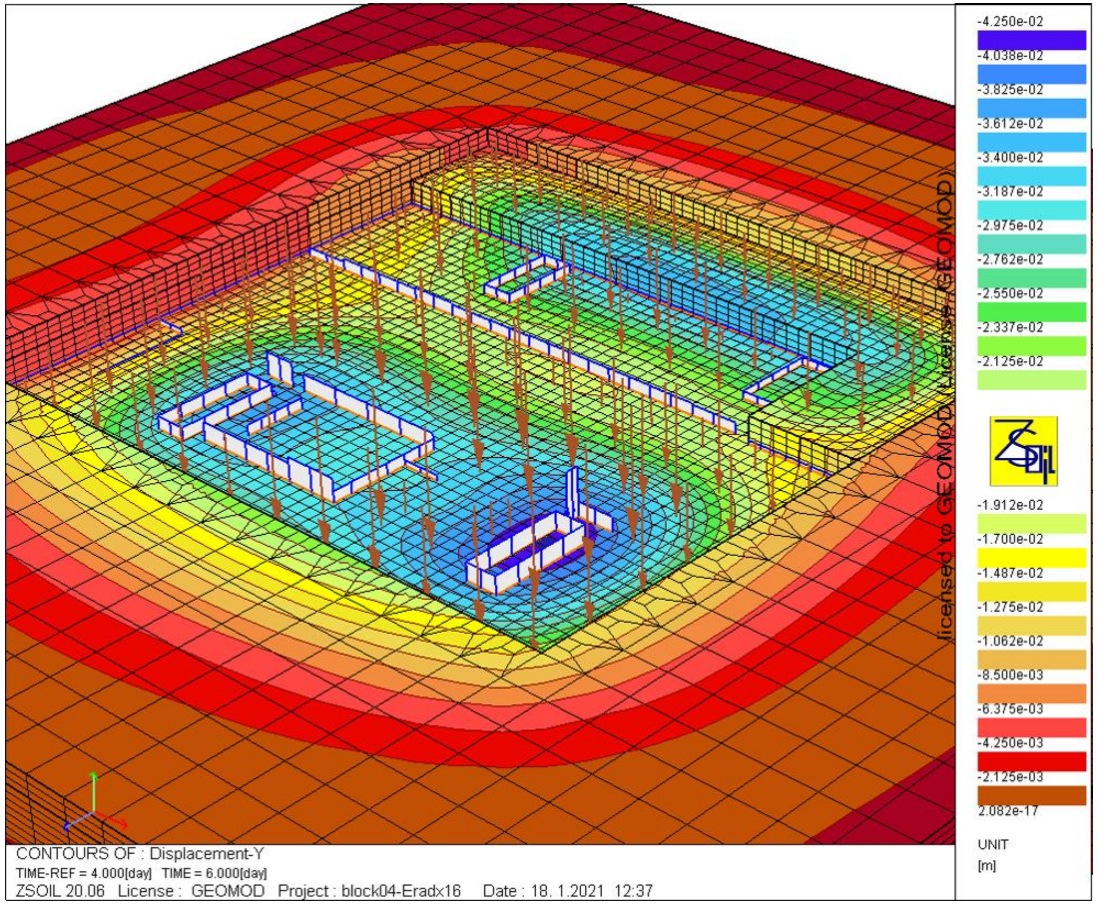
Analysis type	Nr. of samples	ϵ_{LOO}	$UX_{threshold}$ = 40 mm
Numerical MCS	500	-	1.1600e-1
Numerical AK-MCS	30	-	1.1923e-1
Numerical PCE MCS	100'000, built on 10	2.588e-4	1.1158e-1
Numerical PCE MCS	100'000, built on 50	5.645e-5	1.1722e-1
Numerical PCE MCS	100'000, built on 100	1.452e-5	1.1745e-1
Numerical PCE MCS	100'000, built on 100	2.469e-5	1.1754e-1



Example 2: 3D settlement analysis of a concrete foundation

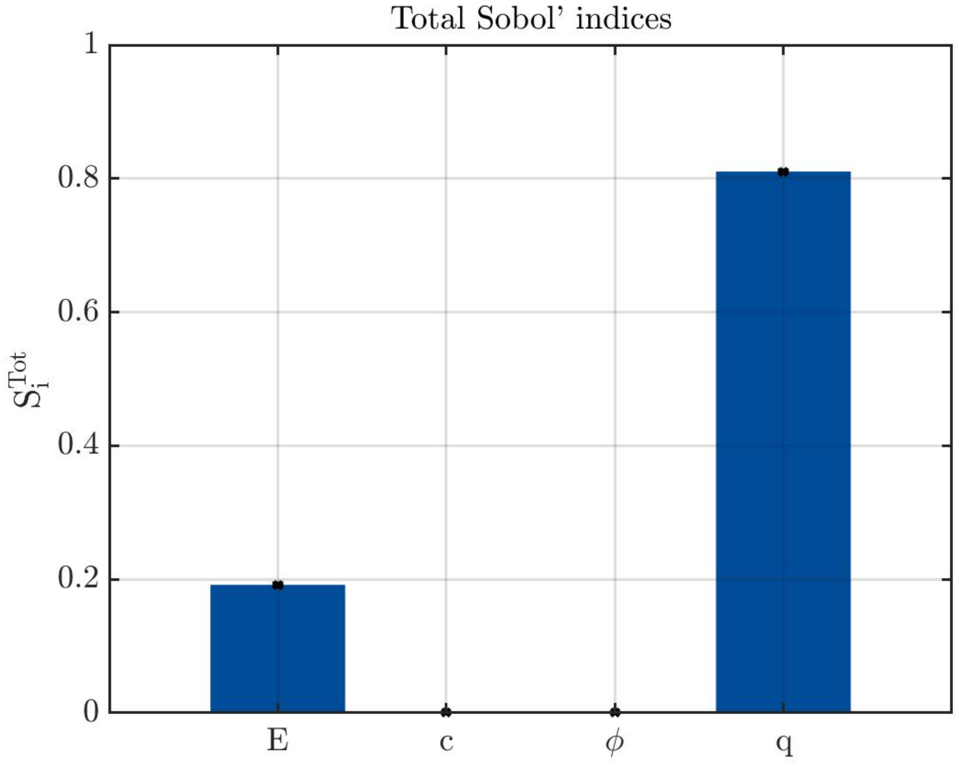
- The settlement of a 96 x 102 m concrete foundation raft that supports a 53'725 squared meters business center to be constructed in the Geneva region in saturated soft silty clays is analyzed here
- A 3D coupled finite element model is constructed with ZSOIL, composed of approx. 60’000 3D brick element (one run = 1 hour)
- The HSS-Brick constitutive model is used in order to represent the soil’s behavior with probabilistic variables E, c, ϕ defined below
- Uncertainty is also introduced on loads q
- Limit-state: g(X): 50 mm – uy(max) < 0

Variable	Description	Distribution	[Lower bound, Upper bound]
E	Young's modulus of soft silty clay	Lognormal	[6'000 kPa, 8'000 kPa]
c	Cohesion of soft silty clay	Lognormal	[5 kPa, 9 kPa]
ϕ	Friction angle of soft silty clay	Lognormal	[24°, 26°]
q	Multiplication factor of the loads	Gumbel	[0.9, 1.1]

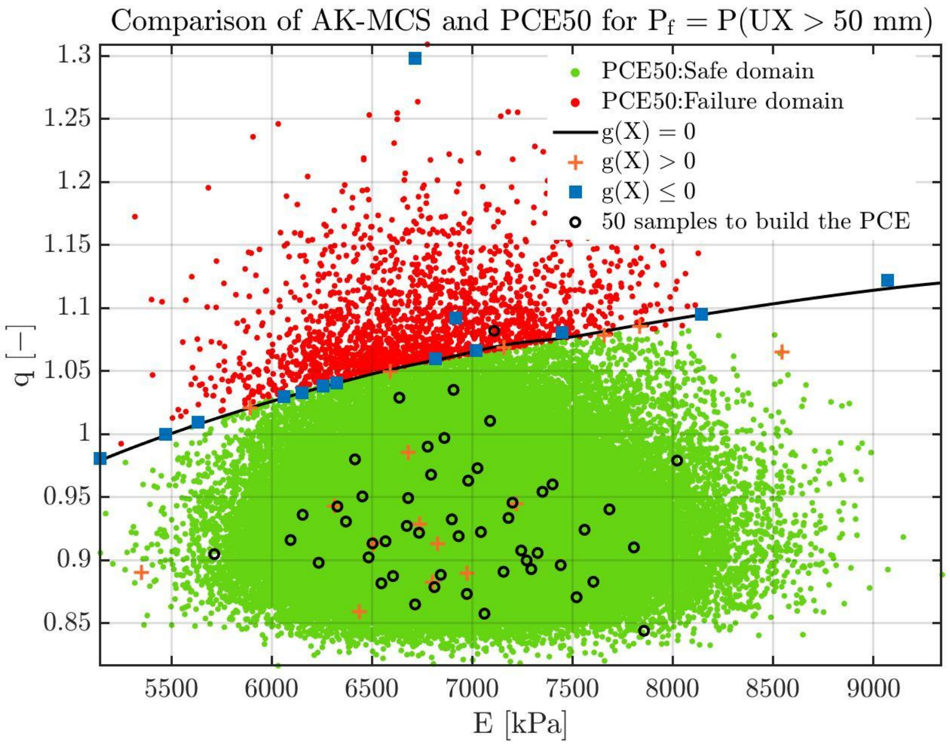


Example 2: 3D settlement analysis of a concrete foundation

Sensitivity analysis

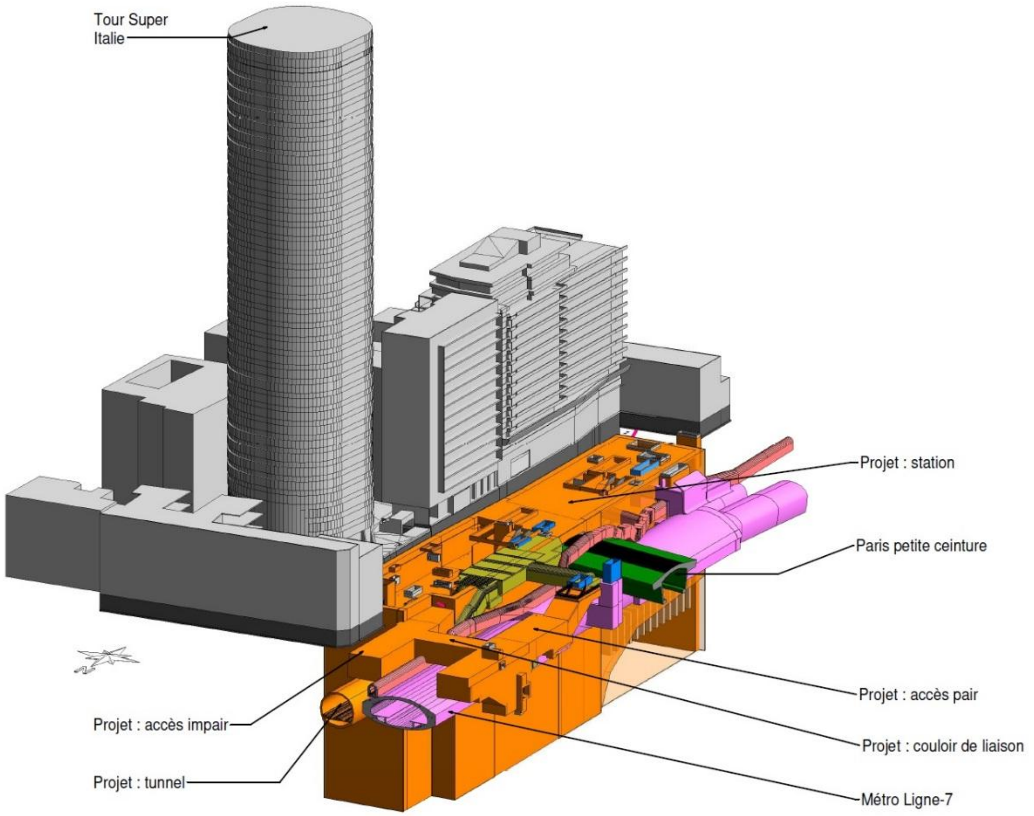


Reliability: Pf = 0.019



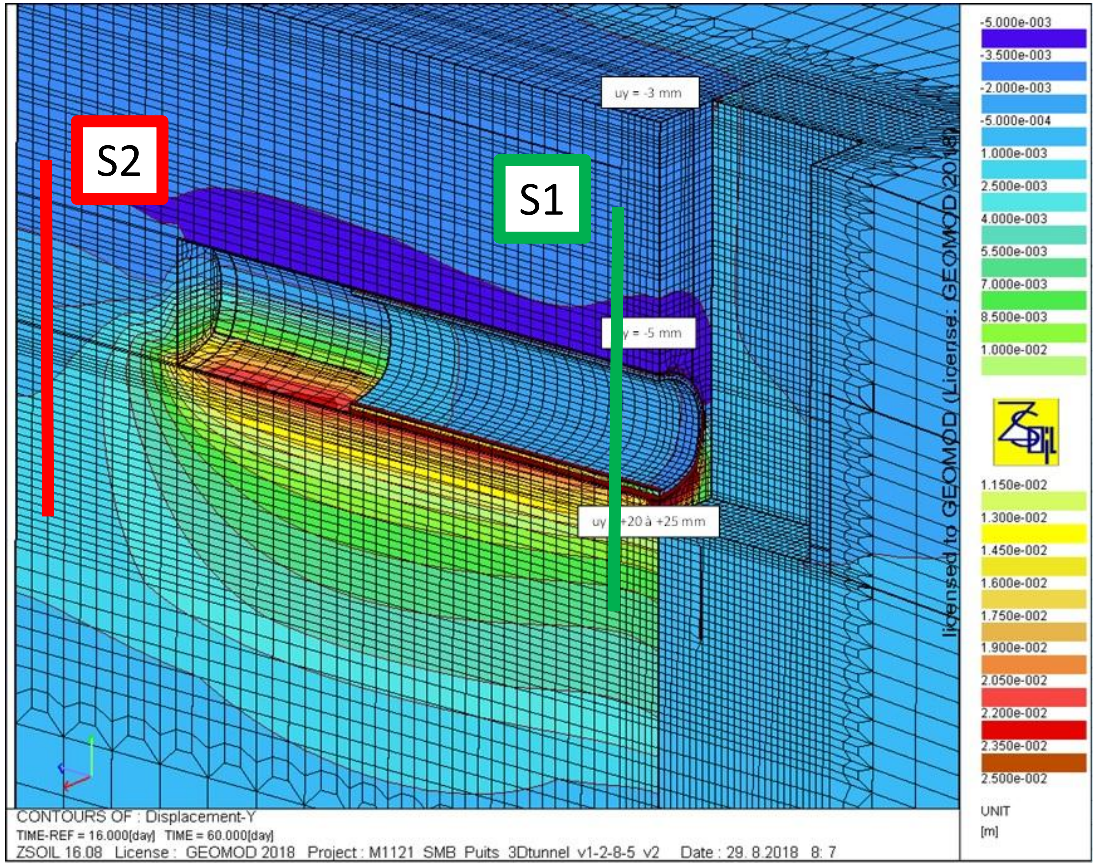
Example 3: inverse analysis applied to a tunnel excavation in Paris

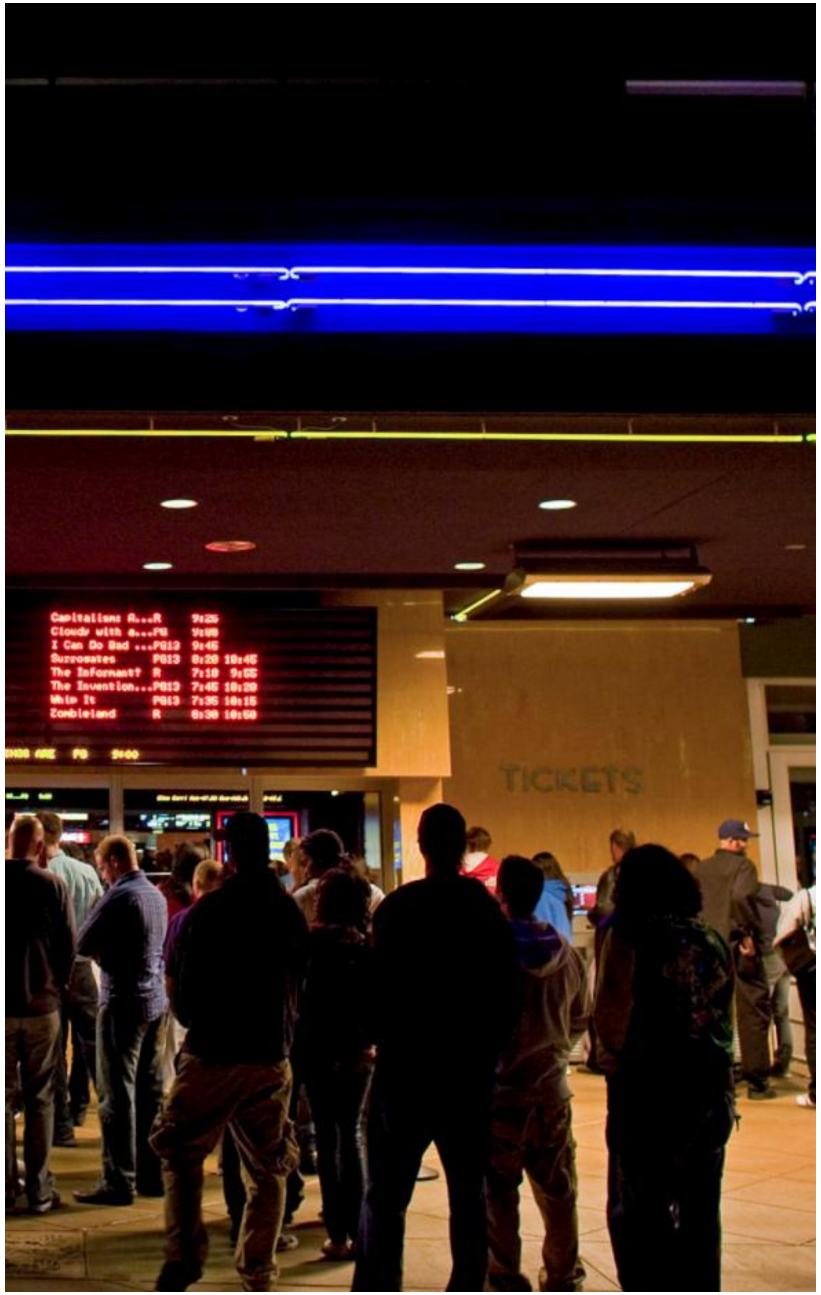
- Extension of L14 between Maison-Blanche and Olympiades
- New station + 150 m tunnel
- What is at stake:
  - Tunnel design stability
  - Swelling in plastic clays
  - Limit settlements on surrounding buildings: threshold = 5 to 10 mm
  - g(X) = threshold - settlement < 0



Example 3: inverse analysis applied to a tunnel excavation in Paris

- Different 2D and 3D FE models have been constructed:
  - 3D shaft + tunnel model
  - 3D tunnel model => estimation of the unloading rate
  - Five 2D cuts across the tunnel
- Question: can we use measurements made at section S1 in order to better predict what will happen at section S2?
- Answer: YES! With the help of Bayesian inverse analysis




Capitalism: A...R 9:25
I Can Do Bad ...PG13 9:45
Surrogates PG13 8:20 10:45
The Informant! R 7:10 9:55
The Invention...PG13 7:45 10:20
Whip It PG13 7:35 10:15
Zombieland R 8:30 10:50
TICKETS

Bayes theorem

- How can we update a prior probability that a hypothesis H holds, based on evidence E?

$$P(H|E) = \frac{P(H) \cdot P(E|H)}{P(E)} = \frac{P(H) \cdot P(E|H)}{P(H) \cdot P(E|H) + P(\neg H) \cdot P(E|\neg H)}$$

$$Posterior\ probability = \frac{Prior\ probability \cdot Likelihood}{Evidence}$$

Bayes theorem

- Example: I have a runny nose, and I have tested positive to CoVid (test is know to be 80% accurate). What is probability that I really have CoVid-19, given that 5% of tested people actually have CoVid?

$$P(Cov|+) = \frac{P(Cov)\cdot P(+|Cov)}{P(Cov)\cdot P(+|Cov)+P(\neg Cov)\cdot P(+|\neg Cov)} = \frac{0.05\cdot 0.80}{0.05\cdot 0.80+0.95\cdot 0.20}$$

$$P(Cov|+) = 17\%$$

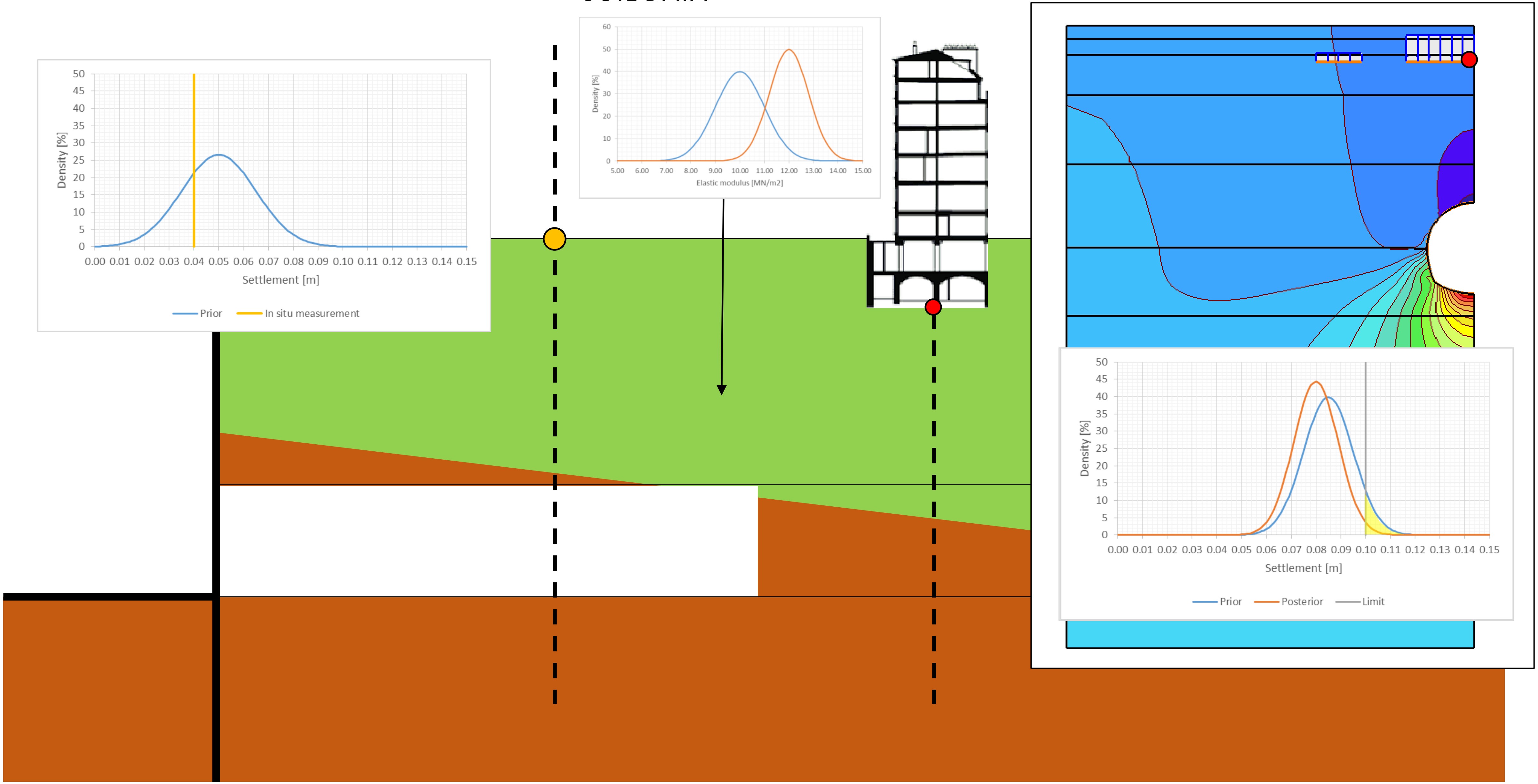
Bayes theorem

- Application to geotechnical engineering: update the prior probability that S > T, given a new measurement M

$$P(S > T|M) = \frac{P(S > T) \cdot P(M|S > T)}{P(M)}$$

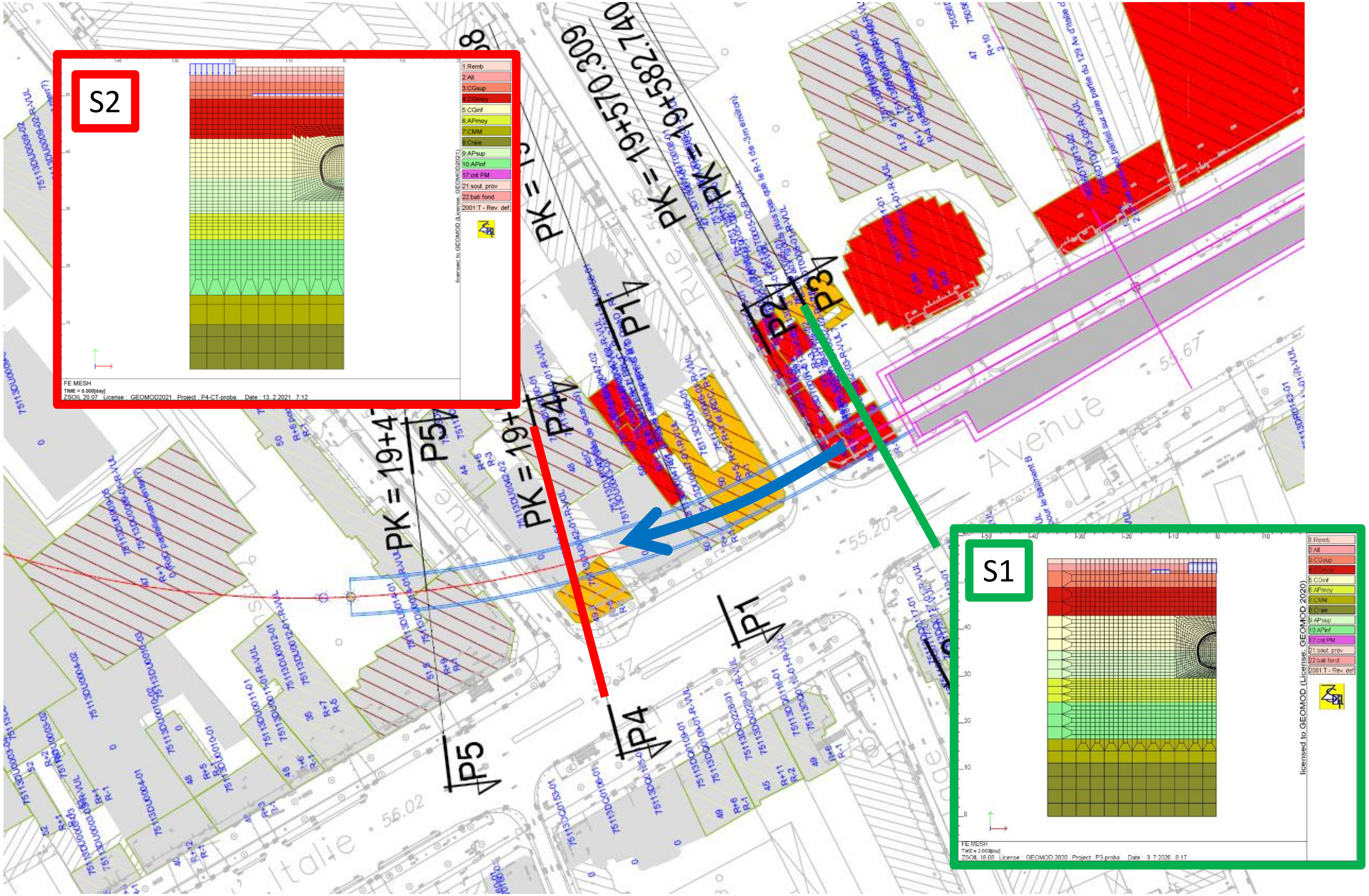
- Remark: not straightforward... we need to use Bayesian inference, that is updating the model's input parameters, given a result (output)

SOIL DATA



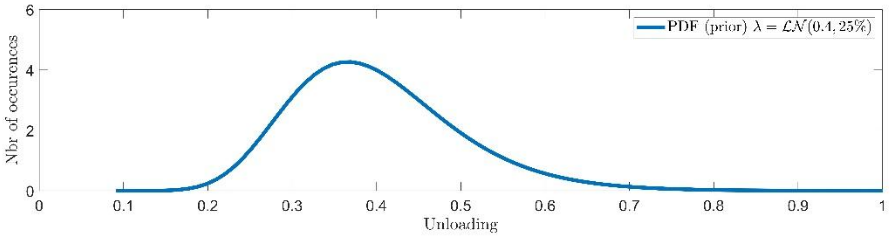
Polish Academy of Sciences
Geotechnical division

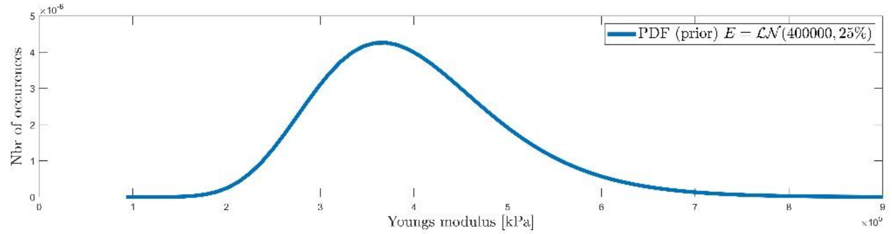
Haute école d'ingénierie et d'architecture Fribourg
Hochschule für Technik und Architektur Freiburg

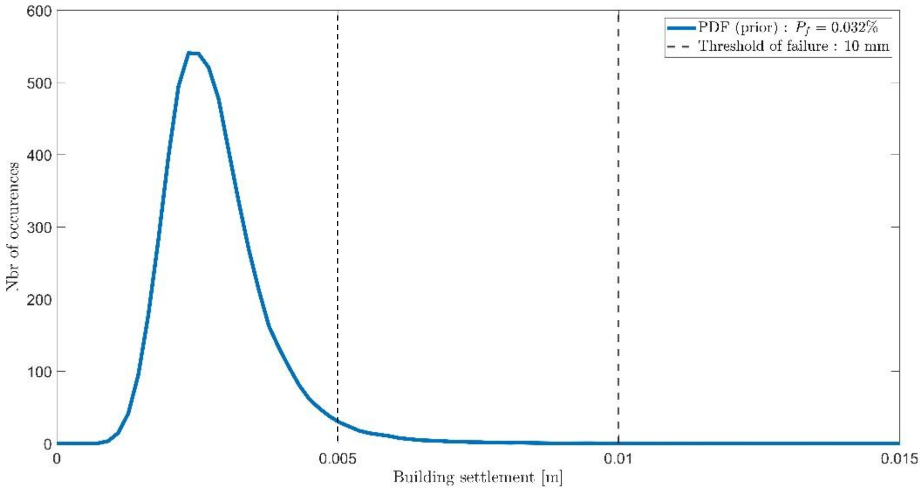

S2
1:Remb
2:All
3:CGsup
4:CGmoy
5:CGinf
6:APmoy
7:CMM
8:Craie
9:APsup
10:APinf
17:cnt PM
21:sout. prov
22:bati fond
2001:T - Rev. def.
FE MESH
TIME = 0.000[day]
ZSOIL 20.07 License : GEOMOD2021 Project : P4-CT-proba Date : 13. 2.2021 7:12
PK = 19+570.309
PK = 19+582.740
P1
P2
P3
P4
P5
Rue
Avenue
55.67
55.20
56.02
Italie
S1
FE MESH
ZSOIL 18.08 License : GEOMOD 2020 Project : P3-proba Date : 3. 7.2020 8:17

Example 3: inverse analysis applied to a tunnel excavation in Paris

- Compute the prior estimate PDF for the building settlement at section S2 with original input distributions for E (400 MPa, COV = 25%) and λ (0.4, 25%)
- For this, we use a PCE surrogating the FEM on 200 samples
- The **prior probability** that the existing building settlement at S2 exceeds 5 mm is $\boldsymbol{P_f}$ **= 2.52%**

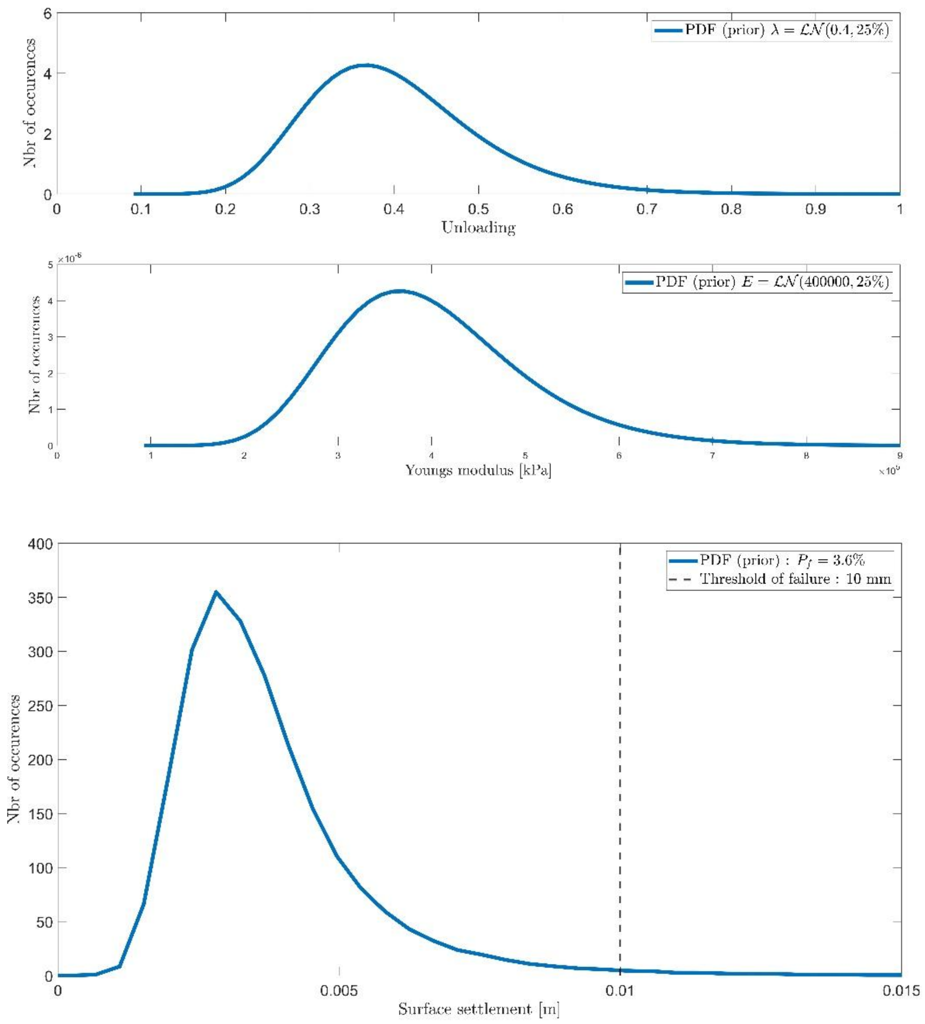






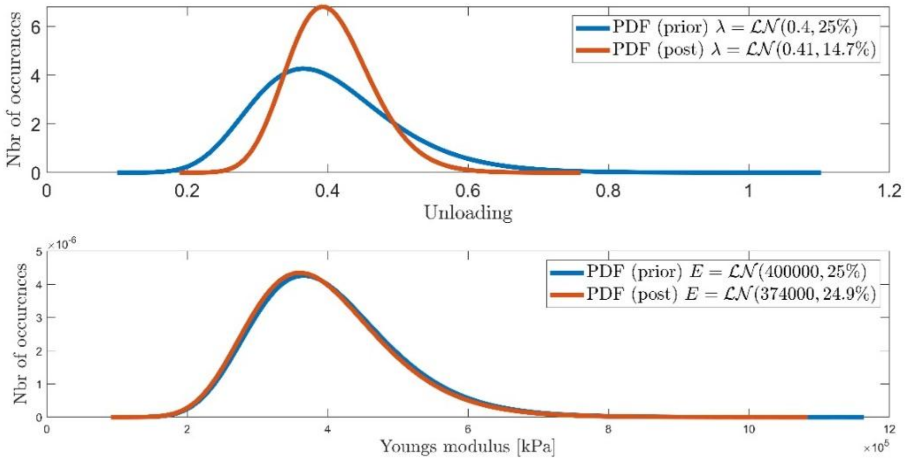
Example 3: inverse analysis applied to a tunnel excavation in Paris

- Compute the prior estimate PDF of the surface settlement at section S1, with original input distributions



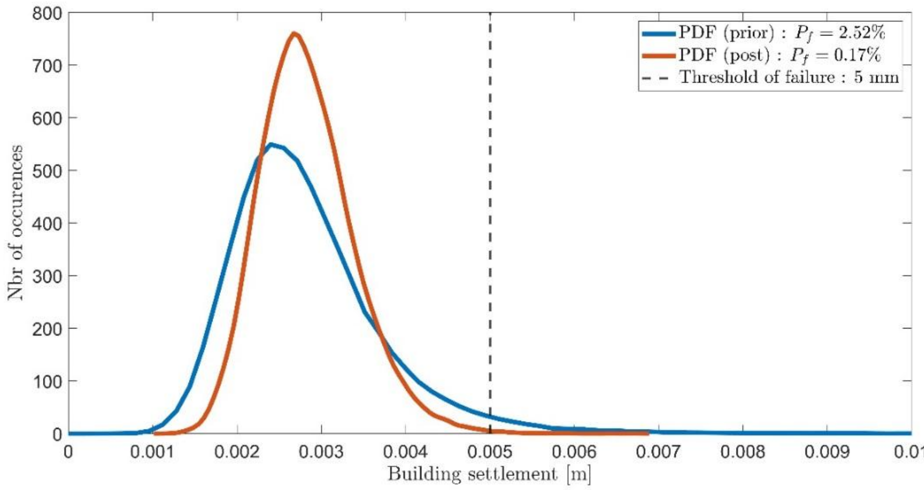
Example 3: inverse analysis applied to a tunnel excavation in Paris

- Assume we measure a 4 mm settlement at section S1, after excavation of the tunnel
- We can use this measurement as an "evidence", in order to update the PDF of the probabilistic input variables E and λ



Example 3: inverse analysis applied to a tunnel excavation in Paris

- Computing the PDF for the building settlement at section S2 with updated input distributions leads to a **posterior probability** that the existing building settlement exceeds 5 mm $\boldsymbol{P_f}$ **= 0.17%** (prior was 2.52%)
- The influence of the COV of the unloading on the probability of failure is here determinant



Conclusion and perspectives

- In geotechnical engineering, sensitivity and reliability analyses give us insight on what the actual risk is
- Deterministic analysis: useful, but not enough, given the uncertainty on the input
- Inverse analysis (Bayesian approach) helps us refine our prior estimates into posterior ones
- Taskgroup now working on guidelines for using probabilistic methods within the new EC7

- Current work @GeoMod & @HEIA-FR: interfacing ZSOIL and UQLab, random fields (spatial variability), reliability-based design optimization

- There is a course given @HEIA-FR (or via Zoom), next edition is Thursday, November 18th, 2021

- Interested? Send us an e-mail!

stephane.commend@hefr.ch or info@geomod.ch

Conclusion and perspectives

- THANK YOU FOR YOUR ATTENTION…

- QUESTIONS?

Acknowledgements

- Prof. Sudret, Dr Marelli, ETHZ
- Prof. Th. Zimmermann, Zace Ltd.
- My colleagues @GeoMod